Optimal trade-off analysis for efficiency and safety in the spacecraft rendezvous and docking problem

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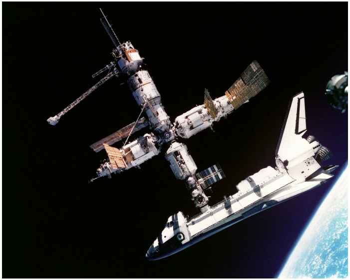
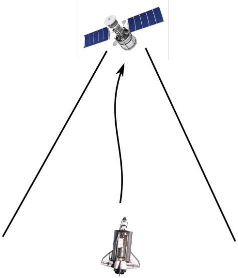
Motivation





- Stochasticity $\leftarrow$ disturbances and unmodeled phenomena
- Stochastic optimal control with requirements of
  - Safety (High probability of state constraints satisfaction)
  - Efficiency (Low fuel consumption)
- Can we maximize safety and efficiency simultaneously?

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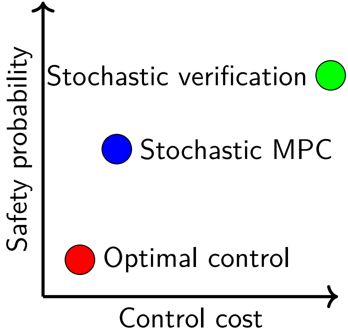

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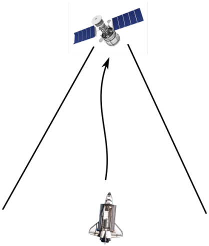
Spacecraft rendezvous and docking problem



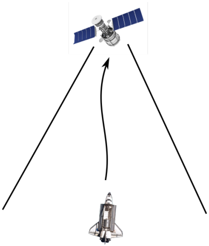
- Two spacecrafts in same circular orbit
- Relative planar dynamics: Clohessy-Wiltshire-Hill

$$\left.\begin{aligned} \ddot{x} - 3\omega x - 2\omega\dot{y} &= \frac{u_1}{m_d} \\ \ddot{y} + 2\omega\dot{x} &= \frac{u_2}{m_d} \end{aligned}\right\} \overset{T_s}{\Rightarrow} \left\{\begin{aligned} \boldsymbol{x}_{t+1} &= A\boldsymbol{x}_t + B\overline{u}_t + \boldsymbol{w}_t \\ \boldsymbol{x}_t &= [x_t\ y_t\ \dot{x}_t\ \dot{y}_t]^\top \\ \boldsymbol{w}_t &\sim \mathcal{N}(\overline{0}, \Sigma_{\boldsymbol{w}}) \end{aligned}\right.$$

Parameter	Symbol	Value
Sampling time period	T_s	$20\ \mathrm{s}$
Orbital radius	R_0	$7.2281 \times 10^6\ \mathrm{m}\ (R_e + 850\ \mathrm{km})$
Gravitational constant times Earth's mass	$\mu = GM_e$	$3.986 \times 10^{14}\ \mathrm{m}^3\mathrm{s}^{-2}$
Orbital frequency	$\omega = \sqrt{\frac{\mu}{R_0^3}}$	$1.027 \times 10^{-3}\ \mathrm{rad\ s}^{-1}$
Deputy spacecraft mass	m_d	$300\ \mathrm{kg}$
Noise covariance	$\Sigma_{\boldsymbol{w}}$	$\mathtt{diag}([10^{-4}\ 10^{-4}\ \frac{10^{-9}}{2}\ \frac{10^{-9}}{2}])$

Lesser, Oishi, & Erwin, CDC 2013

Problem statements



Q1 Maximize probability of staying in line-of-sight cone $\mathcal{S}$, reaching target $\mathcal{T}$ at N; and minimize fuel

Q2 Characterize an empirical lower bound on thruster limits

$$\begin{array}{ll}\underset{\overline{u}_0,\ldots,\overline{u}_{N-1}}{\text{minimize}} & \left[\begin{array}{c} L(\overline{U}) \\ -\mathbb{P}_{\boldsymbol{X}}^{\overline{x}_0,\overline{U}}\{\text{Reach } \mathcal{T} \text{ and stay within } \mathcal{S}\}\end{array}\right] \\ \text{subject to} & \boldsymbol{x}_{k+1} = A\boldsymbol{x}_k + B\overline{u}_k + \boldsymbol{w}_k, \\ & \overline{u}_k \in \mathcal{U} = [-\overline{u}_{\mathrm{bound}}, \overline{u}_{\mathrm{bound}}]^2, \\ & \boldsymbol{w}_k \sim \mathcal{N}(\overline{0}, \Sigma_{\boldsymbol{w}}) \end{array}$$

where $\overline{U} = [\overline{u}_0 \ \ldots \ \overline{u}_{N-1}]$, $L(\overline{U}) = \|\overline{U}\|_2$, $N = 5$ time steps (100 s), $\boldsymbol{X} = [\boldsymbol{x}_1^\top \ \ldots \ \boldsymbol{x}_N^\top]$, $\boldsymbol{X} = \mathscr{A}\overline{x}_0 + H\ \overline{U} + G\ \boldsymbol{W}$, $\boldsymbol{X} \sim \mathcal{N}(\overline{\mu}_{\boldsymbol{X}}, \Sigma_{\boldsymbol{X}})$

Related work

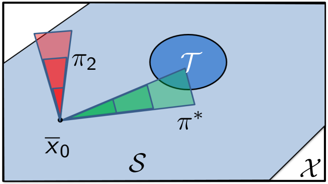
Stochastic verification
Abate, Amin, Prandini, Lygeros, & Sastry (2007); Abate, Prandini, Lygeros, & Sastry (2008); Summers, & Lygeros (2010); Lesser, Oishi, & Erwin (2013); Gleason, Vinod, & Oishi (2017); Kariotoglou, Kamgarpour, Summers, & Lygeros (2017); Vinod & Oishi (2017, 2018)

Bi-criterion optimization
Pareto (1971); Luenberger (1995); Boyd & Vanderberge (2004);

Stochastic MPC approaches
Park, Cairano, & Kolmanovsky (2011); Gavilan, Vazquez, & Camacho (2012); Hartley, Trodden, Richards, & Maciejowski (2012); Weiss, Baldwin, Erwin, & Kolmanovsky (2015); Starek, Schmerling, Maher, Barbee, & Pavone (2016)

Lexicographic approaches
Dueri, Leve, Açıkmeşe (2016); Lesser and Abate (2017)

Verification of LTI+Gaussian via convex optimization



- Reach-avoid objective: $\forall k \in \mathbb{N}_{[0,N-1]},\ \boldsymbol{x}_k \in \mathcal{S} \ \wedge\ \boldsymbol{x}_N \in \mathcal{T}$
- Admissible feedback laws $\mathcal{M} = \{\pi : \mathcal{X} \to \mathcal{U} | \pi \text{ is measurable}\}$

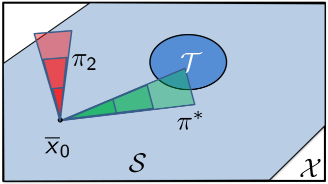
$$\begin{array}{ll} \text{maximize} & \mathbb{P}^{\bar{x}_0,\pi}_{\boldsymbol{X}}\{\text{Reach-avoid}\} \\ \text{subject to} & \pi_k(\cdot) \in \mathcal{M} \end{array} \quad \geq \quad \begin{array}{ll} \text{maximize} & \mathbb{P}^{\bar{x}_0,\overline{U}}_{\boldsymbol{X}}\{\text{Reach-avoid}\} \\ \text{subject to} & \overline{U} \in \mathcal{U}^N \end{array}$$

Dynamic programming
Hard to compute!

Log-concave optimization
Easy to compute!

Vinod & Oishi, LCSS 2017
Abate et. al., Automatica 2008; Summers & Lygeros, Automatica 2010

Verification of LTI+Gaussian via convex optimization



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$$\begin{array}{ll} \text{maximize} & \mathbb{P}_{\boldsymbol{X}}^{\overline{x}_0,\pi}\{\text{Reach-avoid}\} \\ \text{subject to} & \pi_k(\cdot) \in \mathcal{M} \end{array} \quad \geq \quad \begin{array}{ll} \text{maximize} & \int_{\mathcal{S}^{N-1}\times\mathcal{T}} \mathcal{N}(\overline{\mu}_{\boldsymbol{X}}, \Sigma_{\boldsymbol{X}}) \\ \text{subject to} & \overline{U} \in \mathcal{U}^N \end{array}$$

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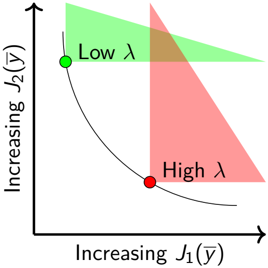
Bi-criterion optimization

$$\begin{aligned} \underset{\overline{y}}{\text{minimize}} \quad & (\text{w.r.t. } \in \mathbb{R}^2_+) \begin{bmatrix} J_1(\overline{y}) \\ J_2(\overline{y}) \end{bmatrix} \\ \text{subject to} \quad & \overline{y} \in \mathcal{Y} \end{aligned} \tag{1}$$

Scalarization: Choose $\lambda \in [0, \infty]$ to convert (1) into (2),

$$\begin{aligned} \underset{\overline{y}}{\text{minimize}} \quad & [1 \;\; \lambda] \begin{bmatrix} J_1(\overline{y}) \\ J_2(\overline{y}) \end{bmatrix} = J_1(\overline{y}) + \lambda J_2(\overline{y}) \\ \text{subject to} \quad & \overline{y} \in \mathcal{Y} \end{aligned} \tag{2}$$



Pareto optimal curve by varying λ

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \preceq \begin{bmatrix} 3 \\ 4 \end{bmatrix} \text{ but } \begin{bmatrix} 1 \\ 2 \end{bmatrix} \not\preceq \not\succeq \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Boyd & Vanderberge, 2004

Trade-off analysis between safety and efficiency

$$\begin{array}{ll}\underset{\overline{U}}{\text{minimize}} & (\text{w.r.t.} \in \mathbb{R}^2_+) \left[\begin{array}{c} \|\overline{U}\|_2 \\ -\log(\mathbb{P}^{\overline{x}_0,\overline{U}}_{\boldsymbol{X}}\{\text{Reach-avoid}\}) \end{array}\right] \\ \text{subject to} & \overline{U} \in \mathcal{U}^N \end{array} \quad (3)$$

- Convex scalarized problem for (3)
  - Log-concave $\mathbb{P}^{\overline{x}_0,\overline{U}}_{\boldsymbol{X}}\{\text{Reach-avoid}\} = \int_{\mathcal{S}^{N-1}\times\mathcal{T}} \mathcal{N}(\overline{\mu}_{\boldsymbol{X}}, \Sigma_{\boldsymbol{X}})$
- Tractable for polytopic $\mathcal{S}, \mathcal{T}$
  - Genz's algorithm → noisy objective → use `patternsearch`
- Initialize by mean trajectory optimization

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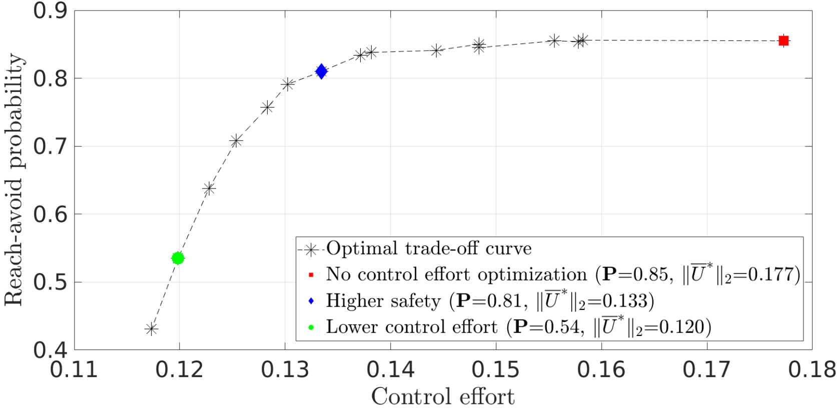
- Tractable for polytopic $\mathcal{S}, \mathcal{T}$
  - Genz's algorithm $\rightarrow$ noisy objective $\rightarrow$ use `patternsearch`
- Initialize by mean trajectory optimization (Quadratic program)

$$\begin{array}{ll}\underset{\overline{\mu}_{\boldsymbol{X}}, \overline{U}}{\text{minimize}} & \|\overline{U}\|_2 \\ \text{subject to} & \overline{\mu}_{\boldsymbol{X}} = \mathscr{A}\overline{x}_0 + \mathscr{H}\ \overline{U} + \mathscr{G}\overline{\mu}_{\boldsymbol{W}}, \\ & P\overline{\mu}_{\boldsymbol{X}} \leq \overline{q}, \\ & H\overline{U} \leq \overline{g}\end{array}$$

Trade-off analysis between safety and efficiency

Initial position (m)	$(0.75, -0.75)$	Input space (N)	$[-0.1, 0.1]^2$
Initial velocity	$(0, 0)$	Compute (min)	~ 59 (17 evals)

Scalarized cost $\lambda\|\overline{U}\|_2 - \log(\mathbb{P}_{\boldsymbol{X}}^{\overline{x}_0,\overline{U}}\{\text{Reach-avoid}\})$, $\lambda \in [0, \infty]$



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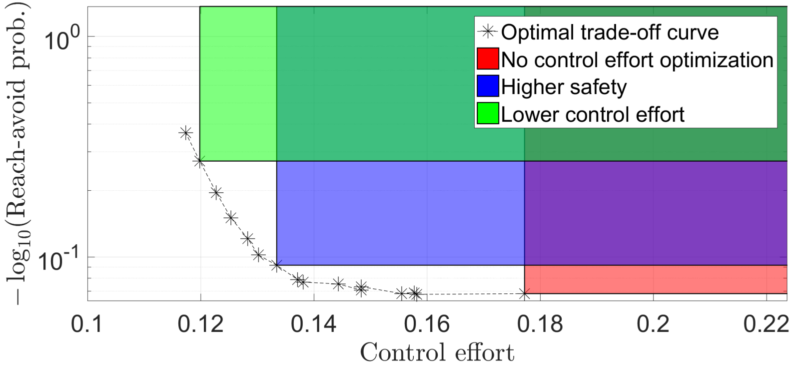

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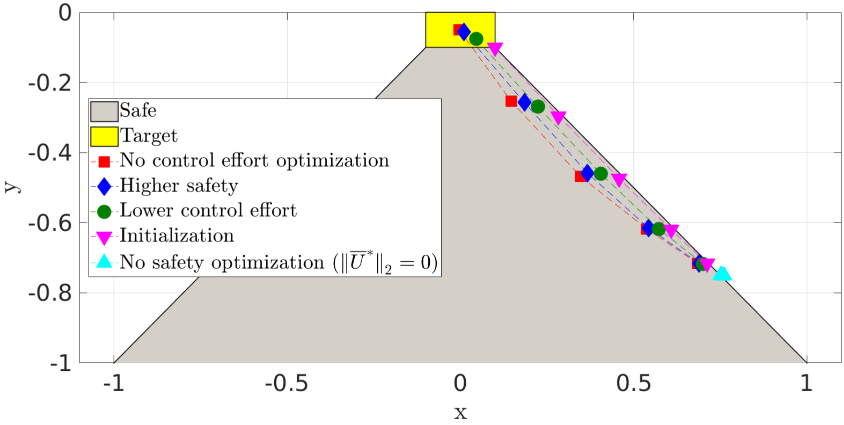
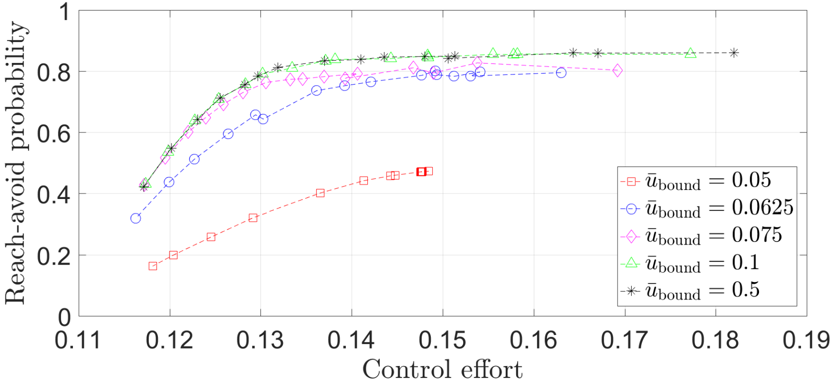


Influence of the control bounds on safety

Initial state $[0.75, -0.75, 0, 0]$

Scalarized cost $\lambda \|\overline{U}\|_2 - \log(\mathbb{P}_{\boldsymbol{X}}^{\overline{x}_0, \overline{U}}\{\text{Reach-avoid}\})$ with $\lambda \in [0, \infty]$

$\mathcal{U} = [-\overline{u}_{\text{bound}}, \overline{u}_{\text{bound}}]^2$ with $\overline{u}_{\text{bound}} \in \{0.05, 0.0625, 0.075, 0.1, 0.5\}$



Summary, future work, and acknowledgements

Summary

- Trade-off analysis b/n safety + efficiency
  - Convex bi-criterion optimization
- Influence of input bounds on safety

MATLAB code: `github.com/unm-hscl/abyvinod-NAASS2018`.

Future work

- Chance-constrained framework
- Analysis of closed-loop controllers
- Linear time-varying system dynamics

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- NSF CMMI-1254990 (CAREER, Oishi),
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